Fall 2011
University of California, Berkeley

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ECONOMICS 210c/ECONOMICS 236a
MACROECONOMIC HISTORY

HANDOUT – SEPTEMBER 14

A Two-Variable VAR

Suppose the true model is:

$$x_{1t} = \theta x_{2t} + b_{11}x_{1,t-1} + b_{12}x_{2,t-1} + \varepsilon_{1t},$$

$$x_{2t} = \gamma x_{1t} + b_{21}x_{1,t-1} + b_{22}x_{2,t-1} + \varepsilon_{2t},$$

where ε_{1t} and ε_{2t} are uncorrelated with one another, with the contemporaneous and lagged values of the right-hand side variables, and over time.

Rewrite this as:

$$\begin{bmatrix} 1 & -\theta \\ -\gamma & 1 \end{bmatrix} \begin{matrix} x_{1t} \\ x_{2t} \end{matrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{matrix} x_{1t} \\ x_{2t} \end{matrix} + \begin{matrix} \varepsilon_{1t} \\ \varepsilon_{2t}, \end{matrix}$$

or:

$$CX_t = BX_{t-1} + E_t,$$

where

$$C \equiv \begin{bmatrix} 1 & -\theta \\ -\gamma & 1 \end{bmatrix}, \; X_t \equiv \begin{matrix} x_{1t} \\ x_{2t}, \end{matrix} \; B \equiv \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}, \; E_t \equiv \begin{matrix} \varepsilon_{1t} \\ \varepsilon_{2t}. \end{matrix}$$

This implies:

$$X_t = C^{-1}(BX_{t-1} + E_t)$$

$$= \Pi X_{t-1} + U_t,$$

where $\Pi \equiv C^{-1}B$, $U_t \equiv C^{-1}E_t$.

Extending to K variables and N lags

The "true model" takes the form:

$$CX_t = \sum_{n=1}^{N} B^n X_{t-n} + E_t,$$

where:

C is K x K,
X is K x 1,
B is K x K,
E is K x 1.

This leads to:

$$X_t = \sum_{n=1}^{N} \Pi^n X_{t-n} + U_t,$$

where $\Pi^n \equiv C^{-1}B^n$, $U_t \equiv C^{-1}E_t$.

Simplified Version of Christiano, Eichenbaum, & Evans

Two variables, one lag:

$$y_t = \qquad b_{11}y_{t-1} + b_{12}r_{t-1} + \varepsilon_{yt},$$

$$r_t = \gamma y_t + b_{21}y_{t-1} + b_{22}r_{t-1} + \varepsilon_{rt}.$$

The reduced form is:

$$y_t = b_{11}y_{t-1} + b_{12}r_{t-1} + \varepsilon_{yt},$$

$$r_t = (b_{21} + \gamma b_{11})y_{t-1} + (b_{22} + \gamma b_{12})r_{t-1} + \left(\gamma\varepsilon_{yt} + \varepsilon_{rt}\right).$$

Simplified Version of Galí

Suppose the true model is:

$$y_t = \theta m_t + b_{11} y_{t-1} + b_{12} m_{t-1} + \varepsilon_{yt},$$

$$m_t = \gamma y_t + b_{21} y_{t-1} + b_{22} m_{t-1} + \varepsilon_{mt}.$$

Long-run impact of a realization of ε_{mt} of +1 on y:

Assume system is such that the impact on *m* eventually settles down at some level; call this level m_{LR}. Then:

$$y_{LR} = \theta m_{LR} + b_{11} y_{LR} + b_{12} m_{LR}.$$

So: $y_{LR} = 0$ requires $\theta + b_{12} = 0$.

Using the restriction that $y_{LR} = 0$ – intuition:

Consider the nonstructural VAR,

$$X_t = \Pi X_{t-1} + U_t.$$

We saw earlier that

$$\begin{matrix} u_{1t} \\ u_{2t} \end{matrix} = \begin{bmatrix} 1 & -\theta \\ -\gamma & 1 \end{bmatrix}^{-1} \begin{matrix} \varepsilon_{yt} \\ \varepsilon_{mt} \end{matrix}$$

$$= \frac{1}{1-\gamma\theta} \begin{bmatrix} 1 & \theta \\ \gamma & 1 \end{bmatrix} \begin{matrix} \varepsilon_{yt} \\ \varepsilon_{mt} \end{matrix}.$$

So a realization of ε_{mt} of +1 raises u_{1t} by $\theta/(1 - \gamma\theta)$ and raises u_{2t} by $1/(1 - \gamma\theta)$.

From the coefficients of the nonstructural VAR, compute long-run effects of unit shocks to each of u_{1t}, u_{2t} on y. Call these S_1, S_2.

Then long-run effect of *m* shock on $y = \theta S_1 + S_2$. Thus, the restriction is $\theta = -S_2/S_1$.

Bernanke and Mihov

$$Y_t \;=\; B_0 Y_t \;+\; \sum_{n=1}^{N} B_n Y_{t-n} \;+\; \sum_{n=1}^{N} C_n P_{t-n} \;+\; A^y v_t^y,$$

$$P_t \;=\; G_0 P_t \;+\; \sum_{n=0}^{N} D_n Y_{t-n} \;+\; \sum_{n=1}^{N} G_n P_{t-n} \;+\; A^p v_t^p.$$

This implies:

$$P_t = \sum_{n=1}^{N} (I - G_0)^{-1} D_n Y_{t-n} + \sum_{n=1}^{N} (I - G_0)^{-1} G_n P_{t-n} + u_t^p,$$

where $u_t^p \;\equiv\; (I - G_0)^{-1} A^p v_t^p$.

The policy block (with time subscripts omitted for simplicity – everything is date t):

$$TR = \cdots - \alpha FF + v^D,$$

$$(TR - NBR) = \cdots + \beta FF + v^B,$$

$$NBR = \cdots + \phi^D v^D + \phi^B v^B + v^S.$$

Example 1: $\phi^D = \phi^B = 0$. Then NBR = … + v^S, so NBR can be used to measure policy shocks.

Example 2: $\phi^D = 1$, $\phi^B = -1$. Then one can show FF =… - $v^S/(\alpha + \beta)$, so FF can be used to measure policy shocks.